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# CONCENTRATION OF DILUTE MIXTURES IN ROTARY THERMAL DIFFUSION COLUMNS: THE THEORY

*A phenomenological theory for the concentration of dilute solutions by a rotary thermal diffusion column is presented, based on the development of a simplified (2-dimensional) model for the rotary column and subsequent application of Ramser's derivation for the «moving-walls» column. The application of the idealised theory to practical apparatus is discussed and an «equivalent annulus width» for the rotary column defined in identical basis as for the static column. The separation equations thus obtained indicate that the performance of the rotary column is strongly dependent on the constancy (perfection) of the annulus width. For perfect apparatus the separation is independent of the speed of rotation and is greater than static separation. For imperfect columns, however, the separation decreases as the speed of rotation increases, and, ultimately, the rotary performance gets poorer than the conventional static apparatus. The limited experimental results available are discussed in terms of the present theory.*

## 1 — INTRODUCTION

The separation of liquid mixtures by thermal diffusion has been the focus of several work since in 1938 CLUSIUS and DICKELL [1] presented the so-called thermogravitational column. The investigation has been mainly concerned with the establishment of a phenomenological theory for the column separation and with the improvement in the column performance (through some modifications in the basic design of the apparatus), both aims running almost concurrently.

Phenomenological theories have been proposed by several authors but the most successful and original are those of FURRY *et al.* [2] and DEBYE [3] which arrive at the same basic «transport equation» although the derivations and range of application are different. Modified theories have been sucessfully applied to modified column-designs with the exception of the rotary column, a prototype apparatus that, due to its promising characteristics has recently attracted the interest of investigators in thermal diffusion.

The reason why no adequate theory for the rotary column exists lies in the fact that the hydrodynamic pattern inside the annulus of such apparatus is unique; while the other types of columns have only one direction of convection (either natural or forced), in a rotary column the natural convection (axial flow) is normal to the forced convection (tangential flow). As a result, the particles streamlines are three-dimensional rather than two-dimensional, the shape of these streamlines being similar to a helically wrapped coil around the column (the angle of inclination, radius of the xy-position and distance between consecutive vertical points being dependent on the x-coordinate, i. e. the flow is not even symmetrical with respect to x (fig. 1).

## 2 — THE REDUCTION OF THE PROBLEM TO TWO-DIMENSIONS

The non-linear partial differential equation that arises from the application of the continuity equation in the absence of chemical reaction [4] to the 3-dimensional flow pattern in a rotary column is of such complexity that the analytical integration «qua tal» is virtually excluded. The difficulty has already been pointed out by ROMERO [5] who suggested the reduction of the problem dimensions and a subsequent application of the mathematical derivation of the 2-dimensional conventional column. In doing so, the separation equations would be formally identical in both rotary and static columns and the design methods devised for the static apparatus could be «translated» to the rotary column.

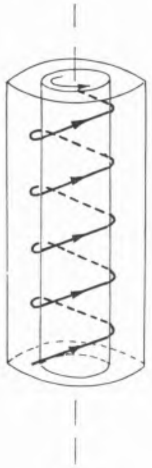

*Fig. 1*
*Particle streamline in the «hot region» of a rotary thermal diffusion column with the inner cylinder rotating*

In his pioneering work, in attempting to correlate his own experimental data, ROMERO [5] derived what he called «an elementary theory» which, in fact, involves a reduction of the problem dimensions. The basic assumption introduced was that the rotary column was physically equivalent to a parallel plate column whose walls move in opposite directions with equal absolute velocities and having the same rate of shear across the annulus as the rotary column. In other words, if the inner cylinder rotates with a tangential velocity V, the equivalent «moving walls» column would have wall velocities of + V/2 and -- V/2. Afterwards, Romero follows a derivation pattern similar to that of FURRY, JONES and ONSAGER [2] for the static column, obtaining a «transport equation» formally identical to the static case but with different transport coefficients. Somewhat surprisingly, though, that author used for the length of the «moving-walls» column the vertical height of the column which is in apparent contradiction with the physical analysis outlined by ROMERO [5] in which it is stated that the length of the particles streamlines (the effective length for cascading) is increased by rotation without, however, changing the residence-time of the particles. As a consequence, the theoretical predictions regarding the steady-state separation are far below the experimental results reported by Romero, but, on the contrary, the theory is in qualitative agreement with the experiment in what concerns the phenomenological parameters that do not depend on the column length.

It may therefore be concluded that the approach of Romero to reduce the problem dimensions, i.e., the physical analysis that led to the assimilation of the rotary column to a moving-walls column is a suitable basis upon which the present theory is to be derived.

Like the conventional theories, the range of validity of the theory presented here is limited by the simplifying assumptions introduced throughout the derivation. Hence, besides the usual limitations [8] it is necessary to consider those involved in the analogy with the «moving-walls» column (related to the speed of rotation) and the range of concentrations under examination (dilute mixtures).

## 3 — THE VELOCITY PROFILE

For the system of fig. 2, the Navier-Stokes equations for the viscous flow of an incompressible newtonian fluid at steady-state hydrodynamic conditions, at speed below the critical Taylor Number, $(N_{Ta})_c$, are [4]

$$\frac{\partial}{\partial z} \rho v_z = 0 \tag{1}$$

$$\frac{\partial p}{\partial x} = 0 \tag{2}$$

$$\eta \frac{\partial^2 v_y}{\partial x^2} = 0 \tag{3}$$

$$\eta \frac{\partial^2 v_z}{\partial x^2} = \frac{\partial p}{\partial z} + \rho g \tag{4}$$

with the following boundary conditions valid for a reservoirless column operated batchwise

$$v_y = V \,, \qquad v_z = 0 \;\text{ at }\; x = + \omega \tag{5}$$

$$v_y = 0\,, \qquad v_z = 0 \quad \text{at} \quad x = -\,\omega \tag{6}$$

$$\int_{-\omega}^{+\omega} v_z dx = 0 \tag{7}$$

The integration of equations (3) and (4) subjected to the above boundary conditions yields

$$v_y = \frac{V}{2}\left(\frac{x}{\omega} + 1\right) \tag{8}$$

$$v_z = \frac{\beta g(\Delta T)\omega^2}{12\eta}\left\{\frac{x}{\omega}\left[1 - \left(\frac{x}{\omega}\right)^2\right]\right\} \tag{9}$$

where it was assumed that $\partial T/\partial x = \Delta T/2\omega$ which is generally valid [7].

The resultant velocity, $\vec{v}_R(x)$, has, thus, a magnitude of

$$\left|\vec{v}_R\right| = \sqrt{v_y^2 + v_z^2} \tag{10}$$

and its deflection angle from the horizontal, $\psi(x)$, is defined through

$$\tan\psi = \frac{y_z}{v_y} \tag{11}$$

It may be easily seen that the angle $\psi$ is at a maximum at $x = +\,\omega/2$ and that the lowest value of $\psi$ in the interval $[+\,\omega, -\,\omega]$ is at $x = -\,\omega$, respectively:

$$\tan\psi\,\Bigg|_{x = \frac{\omega}{2}} = \frac{1}{24}\;\frac{\beta g(\Delta T)\omega^2}{\eta V} \tag{12}$$

$$\tan\psi\,\Bigg|_{x = -\,\omega} = -\,\frac{1}{3}\;\frac{\beta g(\Delta T)\omega^2}{\eta V} \tag{13}$$

For speeds of notation above a certain value [7] — the lower limit of velocity — the extreme values referred to in equations (12) and (13) are sufficiently close for the following assumption to be acceptable:

$$\tan\psi \simeq (\tan\psi)_{av} = -\,\frac{1}{18}\;\frac{\beta g(\Delta T)\omega^2}{\eta V} \tag{14}$$

In this case, also

$$\left|\vec{v}_R\right| \simeq \left|v_y\right| \tag{15}$$

It is convenient, for simplicity, to express the velocity profile of equation (8) in the symmetrical form

$$v_y = \frac{V}{2}\;\frac{x}{\omega} \tag{16}$$

i.e., to assume that both walls are moving in opposite directions with velocities $+\,V/2$ and $-\,V/2$ [5]. The substitution of equation (8) by equation (16) does not interfere with the validity of the model since what is important in terms of the remixing-cascading effects within the annulus are the relative velocities of the particles and not the absolute velocity of the fluid (providing the flow remains below the critical Taylor Number and above the lower limit of velocity).

If, now a new coordinate system, $xy'z'$, is introduced, obtained from the xyz-system of fig. 2 by rotating it around the x-axis by an angle $\psi_{av}$ given by equation (14), the velocity profile $v_R$ (x) in the new system is simply defined by

$$v_R = v_{y'} = \frac{V}{2}\;\frac{x}{\omega} \tag{17}$$

and the inclination from the $xy'$ plan is zero.

The length of contact between the upwards and

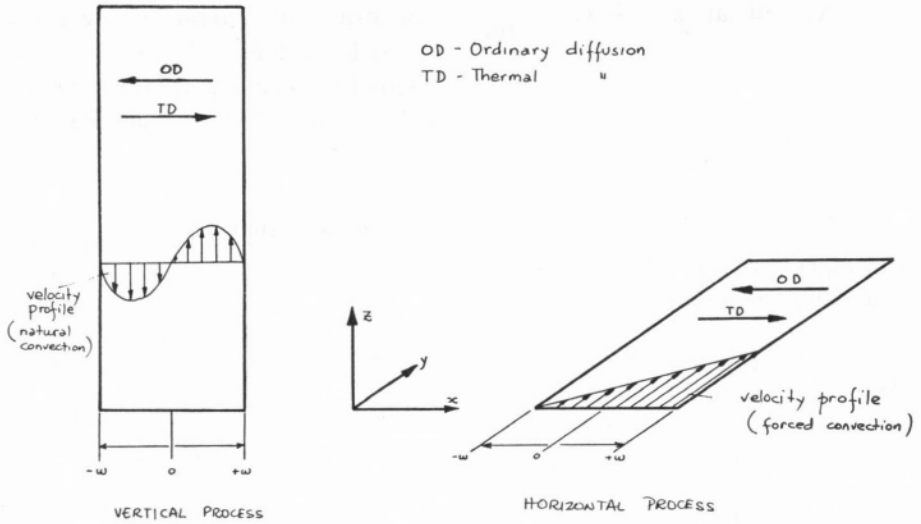

*Fig. 2*
*Model of the rotary thermal diffusion column*

downwards streams (the length of a particle streamline) for a column of vertical height L is, thus, on average

$$L^* = \frac{L}{|(\tan \psi)_{av}|} \tag{18}$$

or, using equation (14),

$$L^* = \frac{L}{|(\tan \psi)_{av}|} = \frac{18\eta V}{\beta g(\Delta T)\omega^2} \tag{19}$$

## 4 — THE MASS FLUXES

In the new coordinate system, xy′z′, the mass fluxes existing inside the rotary column, are ($v_x = v_{z'} = 0$)

$$J_x = -\rho D \frac{\partial c}{\partial x} + \frac{\alpha \rho D}{T_{av}} \frac{\partial T}{\partial x} c(1-c) \tag{20}$$

$$J_{y'} = -\rho D \frac{\partial c}{\partial y'} + v_{y'} c \rho \tag{21}$$

$$J_{z'} = -\rho D \frac{\partial c}{\partial z'} \tag{22}$$

Since the angle $\psi_{av}$ is assumed to be small,

$$\frac{\partial c}{\partial z} \simeq \frac{\partial c}{\partial z'} \tag{23}$$

and, noting that the flow pattern is such that

$$\frac{\partial c}{\partial z} = \frac{L^*}{L} \frac{\partial c}{\partial y'} \tag{24}$$

it is possible taking equations (23) and (24) together to write

$$\frac{\partial c}{\partial z'} = \frac{L^*}{L} \frac{\partial c}{\partial y'} \tag{25}$$

and, also

$$\frac{\partial \rho}{\partial z'} = \frac{L^*}{L} \frac{\partial \rho}{\partial y'} \tag{26}$$

The flux $J_{z'}$ becomes, then

$$J_{z'} = -\rho D \frac{L^*}{L} \frac{\partial c}{\partial y'} \tag{27}$$

and the corresponding term in the continuity equation [4],

$$\frac{\partial(c\rho)}{\partial t} = -\operatorname{div}\vec{J} \tag{28}$$

is, therefore

$$-\frac{\partial J_{z'}}{\partial z'} = \frac{\partial}{\partial y'}\left[\rho D\left(\frac{L^*}{L}\right)^2 \frac{\partial c}{\partial y'}\right] \tag{29}$$

The overall equation is thus

$$\frac{\partial(c\rho)}{\partial t} = -\frac{\partial J_{x'}}{\partial x'} + \frac{\partial}{\partial y'}\left\{\left[\rho D\left(\frac{L^*}{L}\right)^2 + \rho D\right]\frac{\partial c}{\partial y'}\right\} - v_{y'}\frac{\partial(c\rho)}{\partial y'} \tag{30}$$

or, since $L^*/L \gg 1$.

$$\frac{\partial(c\rho)}{\partial t} = -\frac{\partial J_{z'}}{\partial x'} + \frac{\partial}{\partial y'}\left\{\left[\rho D\left(\frac{L^*}{L}\right)^2 \frac{\partial c}{\partial y'}\right\} - v_{y'}\frac{\partial(c\rho)}{\partial y'} \tag{31}$$

Equation (31) is formally identical to that of the more conventional columns for which the treatment of Debye [3] or of Furry *et al.* [2] apply.

## 5 — BATCH SEPARATION EQUATIONS

The reduction of the problem to two dimensions presented in the preceding sections is equivalent to «substituting» the rotary column by a «moving-walls» column whose walls move in opposite directions with velocities $+V/2$ and $-V/2$ and whose length of contact between upwards and downwards streams, $L^*$, is given by equation (19). In this case it is possible to use the results of Ramser [6] for thermal diffusion under linear fluid shear.

Besides the usual simplifying assumptions associated with the derivation of the «fundamental equation» upon which the phenomenological theories for thermal diffusion columns are based, fully discussed by Romero [8], the treatment of Ramser [6] for the «moving-walls» column has the following extra-limitations:

1) $c(1-c) \simeq c$ (i.e. dilute solutions) (32)

2) $T_{av} \gg \alpha(\Delta T)$ (i.e. moderate temperature gradients) (33)

3) $V \gg \frac{11\,D}{2\omega}$ (i.e. forced convection ≫ natural convection) (34)

Under the above conditions the mathematical treatment of Ramser arrives at the following solution for the concentration profile in the direction of shear, which converges for large values of the time variable, t:

$$c = c_o\left\{1 + \frac{40\,\Omega}{120 + \Omega^2}\,\frac{qh}{\pi^2(2\omega)}\sum_{n=0}^{\infty}(-1)^n \cdot \frac{\sin\left[(2n+1)\frac{2\omega}{h}\xi\right]}{(2n+2)}\left[1 - e^{-(2n+1)^2 \cdot t/\theta}\right]\right\} \tag{35}$$

where

$$\Omega = \frac{(2\omega)\,V}{D} \tag{36}$$

$$q = \frac{\alpha(\Delta T)}{T_{av}} \tag{37}$$

$$\theta = \frac{h^2}{\pi^2 D}\,\frac{1}{1 + \Omega^2/96} \tag{38}$$

$$\xi = \frac{y - h}{2\omega} \tag{39}$$

$h$ = length of a particle streamline (40)

The degree of separation, $\Delta$, defined as the difference between the top and bottom concentrations, respectively, $c_T$ and $c_B$, involves the difference between the two infinite series obtained from equation (35) by substituting $\xi$ by

$$\frac{h}{2\omega}$$

and zero, respectively. However, after a time $t \geq 0.3\, t_r$ the terms of the series beyond the first can be neglected with an error less than 1 % and, after rearranging the following expression is obtained

$$\Delta = \Delta_\infty \left(1 - k_3\, e^{-t/t_r}\right) \tag{41}$$

with

$$\Delta_\infty = \frac{10 c_o \alpha D(\Delta T) h}{T_{av}(2\omega) 2V} \tag{42}$$

$$t_r = \frac{96 D h^2}{\pi^2 (2\omega)^2 V^2} \tag{43}$$

$$k_3 = \frac{8}{\pi^2} \tag{44}$$

Introducing the dimensionless length, $\lambda$, defined by

$$\lambda = \frac{504 \alpha D \eta L}{\beta g T_{av}(2\omega)^4} \tag{45}$$

and substituting h which in this case is equivalent to $L^*$ by its value given in equation (19) the following equations are obtained

$$\Delta_\infty = 1.43 \lambda c_o \tag{46}$$

$$t_r = 3.44 \cdot 10^5 \frac{D \eta^2 L^2}{\pi^2 \beta^2 g^2 (\Delta T)^2 (\omega)^6} \tag{47}$$

It is interesting to note that the steady-state separation, $\Delta_\infty$, and the relaxation time, $t_r$, of the rotary column defined by equations (46) and (47) may be expressed in terms of the corresponding static parameters, $(\Delta_\infty)_{st}$ and $(t_r)_{st}$, as

$$\Delta_\infty = 1.43 (\Delta_\infty)_{st} \tag{48}$$

$$t_r = 1.37 (t_r)_{st} \tag{49}$$

where

$$(\Delta_\infty)_{st} = \lambda c_o \tag{50}$$

$$(t_r)_{st} = \frac{9! D \eta^2 L^2}{\pi^2 \beta^2 g^2 (\Delta T)^2 (2\omega)^6} \tag{51}$$

The identical functional dependence of the separation of the rotary and static columns emphasised by equations (48) and (49) is somewhat surprising since it is believed, according to experimental reports, that separation and relaxation-time are affected by rotation. Another interesting observation is that equation (48) predicts that rotation increases the steady-state separation attainable, which has not been always observed in the non-dilute concentration range [5].

It is anticipated that the above conclusions, based on a model derived for an «ideal» situation which may be widely different from the «real» conditions encountered in practice [12], are strongly affected by non-ideal conditions.

## 6 — APPLICATION OF THE «IDEAL» THEORY TO «REAL» COLUMNS

The principal factor affecting the agreement between predicted and experimental results in the conventional static columns has been the non-constancy of the annulus width, due mainly to the eccentricity (either local or along the entire column length). The problem has been the focus of study of several workers [9-11] and, ultimately, BOTT and ROMERO [12] introduced the concept of «equivalent annulus width» based on the physical interpretation of the role played by the eccentricity in altering the fluxes within the column. Briefly, it can be said that the eccentricity of the annulus of a static column promotes and extra-remixing that decreases the separation as if the column had a larger annulus width. The concept was used recently [14] to correlate the experimental results obtained in a commercial column and proved to account for the irregularities in the annular space over the entire separation range.

The effect of eccentricity when the column rotates is qualitatively the same — increase in remixing — but the «equivalent annulus width» must, in principle, be different from the static one since the flow pattern is also different. In fact, in a static column, the particles streamlines are practically parallel to the column walls, whereas for a rotary column (at speeds above the «lower limit») the angle between the particles streamlines and the vertical direction is almost 90 degrees. As a result, the existence of an eccentricity will induce a somewhat «pulsating» remixing effect similar to that of the static column but repeated ($L^*/L$) times. Thus, the equivalent annulus width for a rotary column will be different from that of the static column and, in principle, greater. Also, it is expectable that the speed of rotation will affect strongly the value of the equivalent annulus width of the rotary apparatus since the angle between the particles streamlines and the walls depends on the speed of rotation (and, thus, depends the «pulsating» effect).

If, then ($2\omega$) in the previous equations is considered the static equivalent annulus width as defined by Bott and Romero [12], the «rotary equivalent annulus width», ($2\omega^*$), must be

$$2\omega^* = \frac{2\omega}{\gamma} \tag{52}$$

$$\text{with } \gamma \leq 1 \tag{53}$$

Taking equation (52) into account, the «real» expressions for the steady-state separation, $\Delta_\infty$, and relaxation-time, $t_r$, to use in equation (41) become

$$\Delta_\infty = 1.43\gamma^4\lambda c_o \tag{54}$$

or

$$\Delta_\infty = 1.43\gamma^4 \cdot (\Delta_\infty)_{st} \tag{55}$$

and

$$t_r = 3.44 \cdot 10^5\gamma^6 \frac{D\eta^2L^2}{\pi^2\beta^2g^2(\Delta T)^2(2\omega)^6} \tag{56}$$

or

$$t_r = 1.37\gamma^6 \cdot (t_r)_{st} \tag{57}$$

With equations (55) and (57) it is now seen that the improvement or decrease in separation relatively to the static column will depend essentially on the actual value of $\gamma$, that is, on the constancy of the annulus width and speed of rotation.

A means of testing the validity of the theory and the adequacy of the parameter $\gamma$ just introduced is to determine the steady-state separation and relaxation-time experimentally and compare the values of $\gamma$ evaluated through equations (55) and (57) that carry distinct functional dependences on $\gamma$.

## 7 — PUBLISHED EXPERIMENTAL WORK AND THE THEORY

Although the concentration of dilute solutions by conventional thermal diffusion has been often referred in the literature, the use of a rotary column in this range of concentration has only been reported by Bott [13] who worked with a dilute aqueous solution of glycerol (5 % mole per. glycerol).

Bott studied the influence of the speed of rotation on separation but restricted it to the first phase of the transient period of separation where $\partial\Delta/\partial t$ is larger (this phase has the greater potential interest for large-scale application). The results of Bott [13] are, therefore, of limited value to a complete test of the theory. Notwithstanding, the experimental separation curves obtained, shown in fig. 3, allow some inferences:

(1) The separation is increased by rotation which,

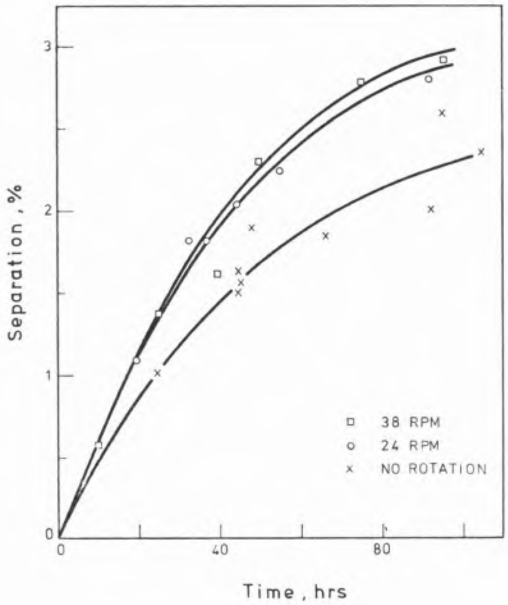

*Fig. 3*
*Separation as a function of time for 5 % glycerol-water system [12]*

in terms of the theory means that $\gamma$ is close to unity or, at least, $(1.43\ \gamma^4) > 1$.

(2) The separation curves for the two speeds of rotation, 24 and 38 RPM, are not significantly deviated which again is an indication that the column is not much imperfect, i.e. that $\gamma$ is close to unity.

As far as the equilibrium-separations and relaxation-times involved are concerned, it is necessary to extrapolate the results of fig. 3 to greater times. This extrapolation may involve a large error and is therefore severely limited in terms of confirming or rejecting the theory. Nevertheless it is worthwhile to estimate, though roughly, the orders of magnitude of the extrapolated and calculated parameters since it provides an example of the type of determinations involved.
By a least squares method it is possible to define the parameters $\Delta_\infty$ and $t_r$ that give the best fit between equation (41) and the experimental points:

$$(\Delta_\infty)_{st} = 0.028$$

$$(\Delta_\infty)_{24} \simeq (\Delta_\infty)_{38} \simeq 0.033$$

$$(t_r)_{st} \simeq 60 \text{ hrs}$$

$$(t_r)_{24} \simeq (t_r)_{38} \simeq 55 \text{ hrs}$$

The value of $\gamma$ may now be evaluated through either equation (55) or (57) aud if the theory is correct those values — referred to as $\gamma_\Delta$ and $\gamma_t$ — should be equal

$$\gamma_\Delta = \left[\frac{0.033}{(1.43)(0.028)}\right]^{\frac{1}{4}} = 0.95$$

$$\gamma_t = \left[\frac{55}{(t.37)(60)}\right]^{\frac{1}{6}} = 0.94$$

If the extrapolated values of $\Delta_\infty$ and $t_r$ are assumed to be valid, the above results indicate that the column is almost perfect ($\gamma \sim 1$) and the test of the theory, positive ($\gamma_\Delta \simeq \gamma_t$).

## 8 — CONCLUSIONS

By comparing the rotary column to a «moving-walls» column with an equal rate of shear across the annulus and defining an «effective length of cascading», it is possible to use the general derivation pattern of the «moving-walls» theory to study the rotary column. The principal results thus obtained may be summarised as follows:

1) Apart numerical factors the separation equations of a rotary column for the concentration of dilute mixtures ($c\ (1 - c) \simeq c$) are identical to the corresponding static equations when the column is geometrically perfect.

2) For a rotary column not geometrically perfect it is necessary to define an equivalent annulus width by $2\omega^* = 2\omega/\gamma$, in which $(2\omega)$ is the static equivalent annulus width as defined by BOTT and ROMERO [12] and $\gamma$ is a correction factor less than unity that may be evaluated experimentally.

3) The separation and relaxation-time are more favourable in the rotary column (better performance), providing that the value of $\gamma$ is close to unity, i.e. that the column is near perfect.

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## *LIST OF SYMBOLS*

$c$ — molar fraction of specified component
$c_o$ — initial feed composition
$c_T$ — composition at the top of the column
$c_B$ — composition at the bottom of the column
$D$ — mutual diffusion coefficient
$D'$ — thermal diffusion coefficient
$g$ — gravity acceleration
$h$ — length of column (for cascading)
$\vec{J}$ — molar flux
$k_3$ — dimensionless parameter defined by eq. (44)
$k$ — thermal conductivity
$L$ — column length (height)
$N_{Ta}$ — Taylor number, $N_{Ta} = 2\rho^2(2\omega)^3V^2/r_m\eta^2$
$(N_{Ta})_c$ — critical Taylor number $= 3.39 \times 10^3$
$p$ — hydrostatic pressure
$q$ — dimensionless parameter defined by eq. (37)
$r$ — cylinder radius
$T$ — absolute temperature
$t$ — time
$t_r$ — relaxation-time
$V$ — tangential velocity (velocity of the moving wall)
$v$ — velocity
$v_R$ — resultant velocity of the combined natural and forced convection
$x, x', y, y', z, z'$ — coordinate directions

## *GREEK LETTERS*

$\alpha$ — thermal diffusion constant
$\beta$ — temperature coefficient of density
$\gamma$ — correction factor defined by eq. (52)
$\Delta$ — degree of separation ($\Delta = | c_T - c_B |$)
$\Delta T$ — temperature difference between hot and cold walls
$\xi$ — dimensionless vertical coordinate defined by eq. (39)
$\lambda$ — dimensionless length defined by eq. (45)
$\eta$ — viscosity coefficient
$\theta$ — dimensionless time defined by eq. (38)
$\pi$ — 3.141592...
$\rho$ — density of the mixture
$\psi$ — deflection angle of the particles streamlines from horizontal
$\Omega$ — dimensionless parameter defined by eq. (36)
$2\omega$ — annulus width (equivalent annulus width) of static column
$2\omega^*$ — equivalent annulus width of rotary column

## *SUBSCRIPTS*

av — average value
st — static
$\infty$ — steady-state
x, y, z — component identification

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## *RESUMO*

*Apresenta-se uma teoria fenomenológica aproximada para descrever a concentração de soluções diluídas numa coluna de difusão térmica rotativa. A teoria baseia-se na construção de um modelo bidimensional simplificado e subsequente aproveitamento da metodologia de Ramser no tratamento de colunas «de paredes móveis». Discute-se a aplicação do modelo teórico a colunas reais, definindo-se, por analogia com a coluna termogravitacional, uma «espessura equivalente» de coluna rotativa. As equações de separação assim obtidas mostram que o rendimento da coluna rotativa depende apreciavelmente da constância (perfeição geométrica) do espaço anular. Para colunas perfeitas o grau de separação é praticamente independente da velocidade de rotação, sendo superior ao grau de separação que se obtém em idênticas condições na coluna estática. Para colunas não perfeitas, contudo, o grau de separação diminui à medida que a velocidade de rotação aumenta, tornando-se, acima de determinada velocidade, inferior ao da coluna estática. Os (poucos) resultados experimentais existentes são discutidos em termos da presente teoria.*